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ON A PARTICULAR CASE OF STATIONARY HEAT TRANSFER WITH CROSS FLOW OF HEAT AGENTS

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Аннотация—Рассматривается задача о двухкратном пространственном перекрёстном токе (Рис. 1). Процесс теплообмена описывается уравнением (1) и граничными условиями (4). Решение выполнено методом интегральных преобразований, и результаты представлены в виде общепринятой зависимости $\psi = f(P, R)$ (Рис. 2).

NOMENCLATURE

$\theta''' = \dfrac{t''' - t'_{in}}{t'''_{in} - t'_{in}}$, dimensionless temperature of a heat agent passing twice in the first passage;

t''', variable temperature of this heat agent in the first passage;

t'', variable temperature of this heat agent in the second passage;

t', variable temperature of the other heat agent;

$U_x = \dfrac{kF_x}{W_3}$, dimensionless co-ordinate along X-axis;

$V_y = \dfrac{kF_y}{W_1}$, dimensionless co-ordinate along Y-axis;

F_x, F_y, heat transfer surfaces counted off along X- and Y-axes respectively;

k, heat transfer coefficient;

W_3, W_1, water equivalents of heat agents passing twice and once in the apparatus;

λ, see formula (7);

p, operator;

F, total heat-transfer surface;

U, V, values of dimensionless co-ordinates at $F_x = F_y = F$;

ν, root of characteristic equation (11);

Δt_{av}, average temperature difference in the apparatus;

$\theta'' = \dfrac{t'' - t'_{in}}{t'''_{in} - t'_{in}}$, dimensionless temperature of a heat agent passing twice in the second passage;

$P = 1 - \theta''_{av}$;

$R = W_3/W_1$;

ψ, correction coefficient for average temperature difference at counter flow.

Subscripts

in, value of quantity at the entrance into the apparatus;

av, average value of quantity at the exit from the apparatus.

In a recuperative heat exchanger, heat transfer between three heat agents is described by a uniform differential equation of the third order when the internal heat source is absent. Here, a particular case of cross motion of the three heat agents at an angle of 90°, shown schematically in Fig. 1, will be considered. Essentially, as is seen from Fig. 1, only two heat agents take part in heat transfer, but one of them, having left the apparatus, enters it again in the opposite direction, assuming that it does not mix in the portion AB.

Thus, this heat agent may be considered as two, namely, a heat agent moving in chamber III, and a heat agent in chamber II. As for heat agent I, it moves normal to the surface plotted in Fig. 1. Separate jets are conventionally marked by points. To the best of the author's knowledge this problem of cross flow is not dealt with in the literature.

In addition to the above-mentioned considerations, we proceed from the following simplifying restrictions and assumptions:

(1) Heat-transfer surfaces separating liquids are flat and equal between themselves.
(2) Heat-transfer coefficients and water equivalents are correspondingly averaged in both passages and, consequently, constant at any point on a heat-transfer surface and at any section of the apparatus.
(3) Liquids moving in the apparatus are not mixed normal to motion.
(4) Heat conduction of a liquid is so small that heat transfer in the direction of its motion may be neglected.
(5) The heat-transfer process is stationary.

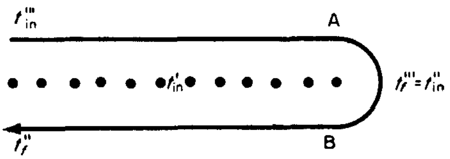

FIG. 1. Scheme of twofold space cross flow.

From these assumptions and conditions, the differential equation describing a process conforming to liquid III may be written as follows:

$$\frac{\partial^3\theta'''}{\partial U_x^2 \partial V_y} + 2\frac{\partial^2\theta'''}{\partial U_x^2} - \frac{\partial\theta'''}{\partial V_y} = 0 \qquad (1)$$

where

$$\theta''' = \frac{t''' - t'_{in}}{t'''_{in} - t'_{in}}; \quad U_x = \frac{kF_x}{W_3}; \quad V_y = \frac{kF_y}{W_1}. \qquad (2)$$

F_x and F_y are heat-transfer surfaces counted off along the X- and Y-axes, respectively; and W_3 and W_1 are water equivalents of heat agents III and I; the remaining symbols are clear from Fig. 1.

If solution (1) is known, dimensionless temperature of a heat agent occurring in the second passage (lower chamber) may be determined from the relation

$$\frac{t'' - t'_{in}}{t'''_{in} - t'_{in}} = \theta'' = \theta''' + \frac{\partial\theta'''}{\partial V_y} + 2\frac{\partial\theta'''}{\partial U_x} + \frac{\partial^2\theta'''}{\partial U_x \partial V_y}. \qquad (3)$$

Solution (1) must satisfy the following boundary conditions:

$$\left.\begin{aligned} \left.\frac{\partial^2\theta'''}{\partial U_x^2}\right|_{V_y=0} &= -\left.\frac{\partial\theta'''}{\partial U_x}\right|_{V_y=0} \\ &= \theta'''\big|_{V_y=0}; \quad \theta'''\big|_{U_x=0} = 1; \\ \left.\frac{\partial^2\theta'''}{\partial U_x \partial V_y}\right|_{U_x=U} &+ 2\left.\frac{\partial\theta'''}{\partial U_x}\right|_{U_x=U} \\ &+ \left.\frac{\partial\theta'''}{\partial V_y}\right|_{U_x=U} = 0 \end{aligned}\right\} \qquad (4)$$

where U is the value of U_x at $F_x = F$, and F is the heat-transfer surface in one of the passages of the apparatus, equal to half of a total surface.

Using the Laplace–Carson transformation of equation (1) and conditions (4) with respect to V_y, we get:

$$\frac{d^2\bar{\theta}'''}{dU_x^2} - \lambda^2\bar{\theta}''' = 0 \qquad (5)$$

$$\bar{\theta}'''\big|_{U_x=0} = 1;$$

$$\left.\frac{d\bar{\theta}'''}{dU_x}\right|_{U_x=U} + \lambda^2\bar{\theta}'''\big|_{U_x=U} = 0, \qquad (6)$$

where

$$\lambda = \sqrt{\left(\frac{p}{p+2}\right)} \quad \text{or} \quad p = \frac{2\lambda^2}{1-\lambda^2} \qquad (7)$$

p being the operator.

The solution of equation (5) satisfying conditions (6) is

$$\bar{\theta}''' = \frac{\cosh\lambda(U - U_x) + \lambda\sinh\lambda(U - U_x)}{\cosh\lambda U + \lambda\sinh\lambda U}. \qquad (8)$$

In the region of images, equation (3) will be as follows:

$$\bar{\theta}'' = \frac{1+\lambda^2}{1-\lambda^2}\bar{\theta}''' + \frac{2}{1-\lambda^2}\frac{d\bar{\theta}'''}{dU_x}; \qquad (9)$$

and that with regard to (8) gives:

$$\bar{\theta}'' = \frac{\cosh\lambda(U - U_x) - \lambda\sinh\lambda(U - U_x)}{\cosh\lambda U + \lambda\sinh\lambda U}. \qquad (10)$$

The denominator of (10) has purely imaginary roots. Designating $\lambda U = i\nu$, we obtain the equation for determining the roots:

$$\cot\nu = \frac{\nu}{U}. \qquad (11)$$

Values of the first six roots are given in Ref. 1, from where values of the first three roots are taken.

Table 1. *Values of the first three roots of equation* (11)

U	ν_1	ν_2	ν_3
0·2	0·433	3·20	6·31
0·4	0·593	3·26	6·35
0·6	0·705	3·32	6·38
0·8	0·791	3·37	6·41
1·0	0·860	3·43	6·44
1·5	0·988	3·54	6·51
2·0	1·08	3·64	6·58
3·0	1·19	3·81	6·70
4·0	1·26	3·93	6·81

Turning from images to originals, we get, from equation (10),

$$\theta'' = 1 - 2U^3 \times \sum_{j=1}^{\infty} \frac{U \cos \nu_j \left(1 - \frac{U_x}{U}\right) + \nu_j \sin \nu_j \left(1 - \frac{U_x}{U}\right)}{\nu_j(U^2 + \nu_j^2)[U(U+1) + \nu_j^2] \sin \nu_j} \times \exp\left(- \frac{2\nu_j^2}{U^2 + \nu_j^2} V_y\right). \quad (12)$$

At any values of V_y the series in equation (12) quickly converges, and at $U < 4$ three terms of the series give the exact value of θ'' for technical calculations.

If in equation (12) V_y is assumed to be equal to zero, then the expansion of the function e^{-2U} with respect to the roots of equation (11) will be obtained.

Assuming $U_x = 0$ in equation (12), we determine the value of relative outlet temperature of the heat agent in the second passage of the apparatus. This temperature will be a function only of the variable V_y, i.e. it will have various values along this co-ordinate.

After averaging θ'' in the range from 0 to V, at the exit from a heat exchanger, the average temperature of the heat agent passing twice in the apparatus is determined:

$$\theta''_{av} = \frac{1}{V} \int_0^V \theta'' \, dV_y$$

or, substituting θ'' for its values from equation (11) at $U_x = 0$,

$$1 - \theta''_{av} = 2 \frac{U^3}{V} \sum_{j=1}^{\infty} \frac{1 - \exp\left(- \frac{2\nu_j^2}{U^2 + \nu_j^2} V\right)}{\nu_j^4[U(U+1) + \nu_j^2]}. \quad (13)$$

Calculating θ''_{av} at various U and V, it is possible to determine the average difference of temperatures in the apparatus from the relation

$$\Delta t_{av} = (t'''_{in} - t'_{in}) \frac{1 - \theta''_{av}}{2U}. \quad (14)$$

In Fig. 2, curves for determining the correction coefficient for the average difference of temperatures at counter flow, depending on $P = 1 - \theta''_{av}$ and $R = W_3/W_1$, are plotted.

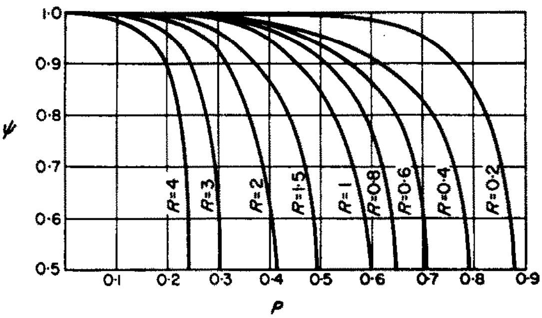

FIG. 2. Dependence of the correction coefficient for the average logarithmic difference of temperatures at the counter flow ψ upon parameters P and R for the scheme considered.

The value of the correction coefficient is determined by the following relation:

$$\psi = \frac{1}{2U(1 - R)} \ln \frac{1 - RP}{1 - P}. \quad (15)$$

The average relative temperature of a less-heated agent having left an inter-tube space is

$$\theta'_{av} = RP. \quad (16)$$

The comparison of curves (Fig. 2) with analogous ones for a single cross flow shows that, in all cases considered, the type of cross flow is characterized by small values of ψ.

REFERENCE

1. A. V. LUIKOV, *Teoria teploprovodnosti*. Gosudarstvennoe Izdatelstvo Tekhnicheskoi Literatury (1952).

Abstract—The problem of twofold space cross flow (Fig. 1) is considered. The heat-transfer process is described by equation (1) and boundary conditions (4). A solution obtained out by the method of integral transformations, and results are given in the form of the generally accepted dependence $\psi = l(P, R)$ in Fig. 2.

Résumé—Cet article étudie le problème de l'écoulement transverse à deux directions. Le processus d'échange thermique est décrit par l'équation (1) et les conditions aux limites par l'équation (4). Une solution est obtenue par la méthode des transformations intégrales et les résultats sont donnés (Fig. 2) sous la forme d'une fonction généralement adoptée, $\psi = 1(P, R)$.

Zusammenfassung—Es wird das Problem des doppelten Querstroms behandelt. Dabei wird das eine Medium nach Durchlauf des Wärmeübertragers umgelenkt und nochmals, wiederum im Querstrom durch den Wärmeübertrager geleitet (Fig. 1). Der Vorgang des Wärmeübergangs ist durch Gleichung (1) und die Randbedingungen (4) beschrieben. Die Lösung wurde nach der Methode der Integral-Transformationen durchgeführt und die Ergebnisse in der Form $\psi = l(P, R)$ in Fig. 2 wiedergegeben.